Drift and Diffusion of carriers

In an electric field, the current of charged carriers is controlled by both drift and diffusion. The sum of these two components will determine how much current will flow.

For electrons:

\[
J_n = q\mu_n n \xi + qD_n \nabla n
\]

For holes:

\[
J_p = q\mu_p p \xi - qD_p \nabla p
\]

\[
J_{\text{cond}} = J_n + J_p
\]

\[
J_n = q\mu_n n \xi + qD_n \frac{\partial n}{\partial x} = q\mu_n \left( n \xi + \frac{kT}{q} \frac{\partial n}{\partial x} \right)
\]

\[
J_p = q\mu_p p \xi - qD_p \frac{\partial p}{\partial x} = q\mu_p \left( p \xi - \frac{kT}{q} \frac{\partial p}{\partial x} \right)
\]
When we put p- and n-doped semiconductors in contact, the Fermi level of both semiconductors has to become equal.

This results in the bending of the conduction and valence bands (band bending) and the establishment of an internal electrostatic potential $V_o$.

When no current is flowing through the junction, $J(\text{drift}) + J(\text{diffusion}) = 0$. 
At equilibrium, when no current is flowing:

\[ J_p(x) = q \left[ \mu_p p(x) \mathcal{E}(x) - D_p \frac{dp(x)}{dx} \right] = 0 \]

Rearranging terms, we get:

\[ \frac{\mu_p \mathcal{E}(x)}{D_p} = \frac{1}{p(x)} \frac{dp(x)}{dx} \]

Substituting \( \mu/D = q/kT \)
(Einstein relationship):

\[ -\frac{q}{kT} \frac{d\mathcal{V}(x)}{dx} = \frac{1}{p(x)} \frac{dp(x)}{dx} \]

And finally, if we integrate, we get:

\[ -\frac{q}{kT} \int_{\mathcal{V}_p}^{\mathcal{V}_n} d\mathcal{V} = \int_{p_p}^{p_n} \frac{1}{p} dp \]

\[ -\frac{q}{kT} (\mathcal{V}_n - \mathcal{V}_p) = \ln p_n - \ln p_p = \ln \frac{p_n}{p_p} \]

This is the built-in potential \( V_o \)
Derivation of the Einstein Relationship:

This relationship can be used to determine the mobility from the diffusion coefficient and vice versa.

\[ J_{\text{drift}} + J_{\text{diff}} = q\mu_n n\zeta + q D_N \frac{dn}{dx} = 0 \]

\[ \zeta = \frac{1}{q} \frac{dE_i}{dx} \] (3.21)

(same as 3.15)

and

\[ n = n_i e^{(E_F - E_i)/kT} \] (3.22)

Moreover, with \( dE_F/dx = 0 \) (due to the positional invariance of the Fermi level under equilibrium conditions),

\[ \frac{dn}{dx} = - \frac{n_i}{kT} e^{(E_F - E_i)/kT} \frac{dE_i}{dx} = - \frac{q}{kT} n\zeta \] (3.23)

Substituting \( dn/dx \) from Eq. (3.23) into Eq. (3.20), and rearranging the result slightly, one obtains

\[ (qn\zeta)\mu_n - (qn\zeta) \frac{q}{kT} D_N = 0 \] (3.24)

Since \( \zeta \neq 0 \) (a consequence of the nonuniform doping), it follows from Eq. (3.24) that

\[ \frac{D_N}{\mu_n} = \frac{kT}{q} \] Einstein relationship for electrons \hspace{1cm} (3.25a)

A similar argument for holes yields

\[ \frac{D_p}{\mu_p} = \frac{kT}{q} \] Einstein relationship for holes \hspace{1cm} (3.25b)
\[ -\frac{q}{kT} \int_{\nu_p}^{\nu_n} d\nu = \int_{p_p}^{p_n} \frac{1}{p} dp \]

\[ -\frac{q}{kT} (\nu_n - \nu_p) = \ln p_n - \ln p_p = \ln \frac{p_n}{p_p} \]

\[ V_0 = \frac{kT}{q} \ln \frac{p_p}{p_n} \]

Since \( np = n_i^2 \),

\[ V_0 = \frac{kT}{q} \ln \frac{N_a}{n_i^2/N_d} = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} \]

\[ \frac{p_p}{p_n} = e^{qV_0/kT} \]
Example: A Si p-n junction has \( N_a = 10^{17} \) on the p side and \( N_d = 10^{16} \) on the n-side. At 300K, what are the Fermi Levels and find \( V_0 \): 

\[
E_{ip} - E_f = kT \ln \frac{p_p}{n_i} = 0.0259 \ln \frac{10^{17}}{1.5 \times 10^{10}} = 0.407 \text{ eV}
\]

\[
E_F - E_{in} = kT \ln \frac{n_n}{n_i} = 0.0259 \ln \frac{10^{16}}{(1.5 \times 10^{10})} = 0.347 \text{ eV}
\]

\[
qV_0 = 0.407 + 0.347 = 0.754 \text{ eV}
\]
There have to be an equal number of charges on either side of the junction. Note that this looks more or less like a capacitor with opposite charges on either side of an insulating region.
Charges must equal on both sides of junction

If it’s a capacitor, the field depends on the dielectric constant and the number of charges (Poisson’s equation)

\[
q Ax_{p0} N_a = q Ax_{n0} N_d
\]

\[
\frac{d\varepsilon(x)}{dx} = \frac{q}{\varepsilon} (p - n + N_d^+ - N_a^-)
\]

Simplifying a bit, we get:

\[
\frac{d\varepsilon}{dx} = \frac{q}{\varepsilon} N_d, \quad 0 < x < x_{n0}
\]

\[
\frac{d\varepsilon}{dx} = -\frac{q}{\varepsilon} N_a, \quad -x_{p0} < x < 0
\]

Next, we integrate to get the field:

\[
\int_{\varepsilon_0}^{0} d\varepsilon = \frac{q}{\varepsilon} N_d \int_{0}^{x_{n0}} dx, \quad 0 < x < x_{n0}
\]

\[
\int_{\varepsilon_0}^{0} d\varepsilon = -\frac{q}{\varepsilon} N_a \int_{-x_{p0}}^{0} dx, \quad -x_{p0} < x < 0
\]

\[
\varepsilon_0 = -\frac{q}{\varepsilon} N_d x_{n0} = -\frac{q}{\varepsilon} N_a x_{p0}
\]
\[ \mathcal{E}_0 = -\frac{q}{\varepsilon} N_d x_{n0} = -\frac{q}{\varepsilon} N_a x_{p0} \]

\[ \varepsilon(x) = -\frac{dV(x)}{dx} \quad \text{or} \quad -V_0 = \int_{-x_{p0}}^{x_{n0}} \varepsilon(x) dx \]

\[ V_0 = -\frac{1}{2} \mathcal{E}_0 W = \frac{1}{2} \frac{q}{\varepsilon} N_d x_{n0} W \]

since \[ \frac{1}{A} x_{p0} N_a = \frac{1}{A} x_{n0} N_d \]

and \( W = x_{p0} + x_{n0} \)

\[ x_{n0} = WN_a/(N_a + N_d), \]

we can substitute:

\[ V_0 = \frac{1}{2} \frac{q}{\varepsilon} \frac{N_a N_d}{N_a + N_d} W^2 \]

This provides us with an equation which we can use to calculate the depletion width if we know the dopant concentrations of a p-n junction.
Depletion width equation:

\[
W = \left[ \frac{2\varepsilon V_0}{q} \left( \frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} = \left[ \frac{2\varepsilon V_0}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2}
\]

where \(N_a\) and \(N_d\) are the Acceptor and Donor Dopant concentrations.

Note: For silicon, \(\varepsilon\), the permittivity, is given as 11.8 x (8.85x10^{-14} \text{ F/cm})

Dielectric constant

Permittivity of free space

We can also determine how much of the depletion width is on the p and n side:

\[
x_{p0} = \frac{WN_d}{N_a + N_d} = \frac{W}{1 + N_a/N_d} = \left\{ \frac{2\varepsilon V_0}{q} \left[ \frac{N_d}{N_a(N_a + N_d)} \right] \right\}^{1/2}
\]

\[
x_{n0} = \frac{WN_a}{N_a + N_d} = \frac{W}{1 + N_d/N_a} = \left\{ \frac{2\varepsilon V_0}{q} \left[ \frac{N_a}{N_d(N_a + N_d)} \right] \right\}^{1/2}
\]
Example: A Si diode is formed from an abrupt junction with $N_d = 10^{16}$ and $N_a = 4 \times 10^{18}$.

Calculate $V_o$, $x_n$, $x_p$, $Q_+$ and the field for this junction.

**Built-in potential $V_o$**

$$V_o = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} = \frac{0.0259 \ln (4 \times 10^{34})}{2.25 \times 10^{20}}$$

$$= 0.0259 \ln (1.78 \times 10^{14}) = 0.85 \, V$$

From Eq. (5–23),

$$x_n = \frac{3.34 \times 10^{-5}}{1 + 0.0025} = 0.333 \, \mu m$$

$$x_p = \frac{3.34 \times 10^{-5}}{1 + 400} = 8.3 \times 10^{-8} \, cm = 8.3 \, \AA$$

Note that $x_n = W$. 

Depletion width

$$W = \left[ \frac{2eV_o}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2}$$

$$= \left[ \frac{2(1.18 \times 8.85 \times 10^{-14})(0.85)(0.25 \times 10^{-18} + 10^{-16})}{1.6 \times 10^{-19}} \right]^{1/2}$$

$$= 3.34 \times 10^{-5} \, cm = 0.334 \, \mu m$$
\[ Q_+ = -Q_- = qAx_{n0}N_d = (1.6 \times 10^{-19})(2 \times 10^{-3})(3.33 \times 10^{-5})(10^{16}) \]

\[ = 1.07 \times 10^{-10} \text{ C} \]

\[ \varepsilon_0 = \frac{-qN_d x_{n0}}{\epsilon} = \frac{-(1.6 \times 10^{-19})(10^{16})(3.3 \times 10^{-5})}{(11.8)(8.85 \times 10^{-14})} \]

\[ = -5.1 \times 10^4 \text{ V/cm} \]
Depletion width $W$ in potential built-in potential permittivity of semiconductor

$W = \left[ \frac{2\varepsilon V_0}{q} \left( \frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} = \left[ \frac{2\varepsilon V_0}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2}$

where $N_a$ and $N_d$ are the Acceptor and Donor Dopant concentrations

Note: For silicon, $\varepsilon$, the permittivity, is given as $11.8 \times (8.85 \times 10^{-14} \text{ F/cm})$

This relationship is for no external voltage applied to the diode. If we apply a reverse bias voltage $V_a$, we add that voltage to $V_o$ and use $(V_a + V_o)$ instead of $V_o$ here.

Note: As we increase the reverse bias voltage, the depletion width increases.
At 0 applied voltage, the current flowing is 0.

At positive voltages, the current follows an exponential dependency on voltage.

At negative voltages, the current saturates to a saturation current $I_{\text{gen}}$.

The Boltzmann diode equation is given by:

$$I = |I(\text{gen.})| (e^{V/kT} - 1)$$

At 0 applied Voltage, the current flowing is 0.
Schematic of the junction terminology used

Excess electrons in the p-material
\[ \delta n(x_p) = \Delta n_p e^{-x_p/L_n} \]

Excess holes in the n-material
\[ \delta p(x_n) = \Delta p_n e^{-x_n/L_p} \]

Depletion width

Position of the metallurgical (diffused) junction
We know that the concentrations of holes in the p side and the concentration of holes on the n side of a p-n junction are related by $V_o$:

$$\frac{p_p}{p_n} = e^{qV_0/kT}$$

Now, we can find the excess minority carrier concentration on both p and n-side of the depletion width by subtracting:

$$\frac{p(-x_{p0})}{p(x_{n0})} = e^{q(V_0-V)/kT}$$

Note that this means that as you apply a positive voltage, the number of excess holes on the n side increases exponentially.

Taking $p(-x_{p0}) = p_p$,

$$\Delta p_n = p(x_{n0}) - p_n = p_n(e^{qV/kT} - 1)$$

$$\Delta n_p = n(-x_{p0}) - n_p = n_p(e^{qV/kT} - 1)$$
Since we know that the excess carrier concentration drops off to zero as we get further away from the junction, we can define diffusion lengths $L_n$ and $L_p$ for electrons and holes, respectively and assume an exponential decay with distance. These diffusion lengths depend on the dopant concentration (recombination).

Usually, the n and p regions are long with respect to $L_n$ and $L_p$ and:

$$\delta n(x_p) = \Delta n_p e^{-x_p/L_n} = n_p \left( e^{qV/kT} - 1 \right) e^{-x_p/L_n}$$

$$\delta p(x_n) = \Delta p_n e^{-x_n/L_p} = p_n \left( e^{qV/kT} - 1 \right) e^{-x_n/L_p}$$

These are the excess hole and electron concentrations as a function of distance $x_n$ and $x_p$ from the junction.
Unlike the muskrat, electrons and holes disappear (recombine) as they diffuse rather than reproduce, but the idea is similar.

The Muskrat again:

The muskrat (Ondatra zibethica) was introduced into Europe in 1905 near Prague. Since that time its area expanded, and the muskrat population moved with the rate ranging from 0.9 to 25.4 km/yr. The intrinsic rate of population increase was estimated at 0.2-1.1 per year, and the diffusion coefficient ranged from 51 to 230 sq.km/yr. Predicted spread rate (6.4-31.8 km/yr) corresponds well to actual rates of spread.

**musk′rat (musk′rat) n., pl. -rats or -rat** [<Algonquian musquash] 1. An aquatic rodent of North America (Ondatra zibethica), having dark, glossy brown fur, a flattened tail, webbed hind feet, and a musky odor; also called water rat. 2. The valuable fur of this rodent.

Note: The Probert E-Text Encyclopaedia makes a distinction between the hyphenated word musk-rat (or musquash) as *Fiber zibethicus* and the unhyphenated muskrat as Ondatra zibethicus. However, Websters Unabridged Dictionary 1913 equates musquash and muskrat. So, (1) there is very little difference in pronunciation between "muskrat" and "musk-rat", and (2) "musquash" may refer to either animal. This casual use of common names may result in references to both Ondatra zibethicus and Fiber zibethicus among my links.
We remember the current density equation, and can simplify it to only contain the diffusion current

\[
J_p(x) = q \left[ \mu_p p(x) \delta(x) - D_p \frac{dp(x)}{dx} \right] = 0
\]

From this, we can define the diffusion current as:

\[
I_p(x_n) = -qAD_p \frac{d\delta p(x_n)}{dx_n} = qA \frac{D_p}{L_p} \Delta p_n e^{-x_n/L_p} = qA \frac{D_p}{L_p} \delta p(x_n)
\]

The total hole current is:

\[
I_p(x_n = 0) = \frac{qAD_p}{L_p} \Delta p_n = \frac{qAD_p}{L_p} p_n(e^{qV/kT} - 1)
\]

The total electron current is

\[
I_n(x_p = 0) = -\frac{qAD_n}{L_n} \Delta n_p = -\frac{qAD_n}{L_n} n_p(e^{qV/kT} - 1)
\]
Finally, the total diode current is the sum of the hole and electron currents across the p-n junction and is given by:

\[ I = I_p(x_n = 0) - I_n(x_p = 0) = \frac{qAD_p}{L_p} \Delta p_n + \frac{qAD_n}{L_n} \Delta n_p \]

\[
I = qA \left( \frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p \right) \left( e^{qV/kT} - 1 \right) = I_0 \left( e^{qV/kT} - 1 \right)
\]

Area of junction  diffusivity  diffusion length  Applied voltage  Temperature (K)

\[ L = \sqrt{D \tau} \]

Can also be substituted for L
For those interested, here are three ways of deriving the diode equation:

\[ I = -qA \left( \frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p \right) = -I_0 \]

\[ I_n(x_p=0) = qAD_n \frac{d\delta n}{dx_p} \bigg|_{x_p=0} = -qA \frac{D_n}{L_n} \Delta n_p \]

\[ I_p(x_n=0) = -qAD_p \frac{d\delta p}{dx_n} \bigg|_{x_n=0} = qA \frac{D_p}{L_p} \Delta p_n \]

\[ \delta n = \Delta n_p e^{-x_p/L_n} \]

\[ \delta p = \Delta p_n e^{-x_n/L_p} \]

\[ Q_n = -qA \int \delta n(x_p) \, dx_p \]

\[ Q_p = qA \int \delta p(x_n) \, dx_n \]

\[ I_n(x_p=0) = \frac{Q_n}{\tau_n} = -\frac{qA L_n}{\tau_n} \Delta n_p \]

\[ I_p(x_n=0) = \frac{Q_p}{\tau_p} = \frac{qA L_p}{\tau_p} \Delta p_n \]

\[ I = I_p(x_n=0) - I_n(x_p=0) = qA \left( \frac{D_p}{L_p} \Delta p_n + \frac{D_n}{L_n} \Delta n_p \right) \]

\[ = qA \left( \frac{D_p p_n}{L_p} + \frac{D_n n_p}{L_n} \right) (e^{qV/kT} - 1) \]
Diode Breakdown Effects:

When we apply a large reverse bias (negative voltage) onto a diode, it eventually breaks down and conducts again. That voltage is called the reverse breakdown voltage $V_{br}$.

There are two common breakdown mechanisms: Tunneling and Avalanche Multiplication.
If a high enough voltage is applied to the depletion region, minority carriers are accelerated through this region and are energetic enough to generate secondary electrons. This results in an "avalanche" effect which amplifies the number of free carriers in the depletion region and results in breakdown.

Note: Avalanche photodiodes are often used as sensitive optical sensors with built-in gain. They are typically biased to 100V.
Plot of breakdown characteristics as a function of dopant concentration. Higher fields occur at small depletion widths and large reverse biases.

For Si, $3 \times 10^5 \text{V/cm}$

$$V_{br} = \varepsilon E_{br}^2 / 2qN_d$$

**Graph Details:**
- GaP, GaAs, Si, Ge
- Depletion widths:
  - Wider depletion widths
  - Narrower depletion widths
- Fields:
  - $300 \text{K}$
  - Tunneling begins

**Equation:**
$$V_{br} = \varepsilon E_{br}^2 / 2qN_d$$
Tunneling or Zener breakdown:

At a high enough voltage, if \( W \) is narrow enough, it is possible for electrons to tunnel from the valence band of the p-side of the diode directly to the conduction band of the n-side of the diode. This results in a very sharp increase in current at the tunneling voltage \( V_T \).

Note: Zener diodes are often used as voltage control devices.

The mechanism for this process is field ionization, which requires very short \( W \) and high fields of \( >10^6 \text{V/cm} \) in the junction. There is not enough distance for impact ionization if \( N_d \) and \( N_a \) are high.

This results in a very sharp increase in current at the tunneling voltage \( V_T \).
Metal semiconductor contacts: p-semiconductor case:

When a metal is deposited on a semiconductor directly the work function of the metal and the Fermi level of the semiconductor must line up again.
Metal semiconductor contacts: n-semiconductor case:

A depletion layer is formed because of the band-bending in this case. The metal behaves very much like a heavily doped p$^+$ layer, and the semiconductor is depleted of electrons.

The work function of a metal is defined as the energy for its (free) electrons to escape into vacuum.

A depletion layer is formed because of the band-bending.
Again, when we apply a voltage, diffusion current will give rise to an exponential increase of the current.

And in reverse bias, the current flow is limited to $I_{gen}$.
For a Metal-Semiconductor (Schottky) diode, the I-V relationship can be described by:

\[ I = A \lambda T^{2} e^{-\frac{q \Phi_B}{kT}} e^{\frac{qV}{nkT}} \]

- **Current**
- **Cross-sectional Area**
- **Barrier height**
- **voltage**
- **ideality factor**

**N** = ideality factor which ranges from 1-2

\[ \Phi_B = \text{Schottky barrier height} \approx 0.85 \text{ for a typical Si surface with a Pt contact} \]

This value depends on the surface of Si and the work function of the metal chosen.

**B** = constant describing the junction properties

Note that the last part of this equation is very similar to the regular p-n diode current equation.
Many semiconductors, such as GaAs have dangling bonds on the surface, and this pins the Fermi level to a fixed value of ~0.8 eV below the conduction band. This results in band bending and surface depletion even if there is no metal on the surface.

In InAs, the Fermi level is pinned above the conduction band edge. This means that this material is ideal for constructing ohmic contacts.